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PAPER ID : 49905	Roll No.										

B. Tech. Examination 2023-24

(Even Semester)

DIFFERENTIAL EQUATIONS AND FOURIER ANALYSIS

Time : Three Hours] *[Maximum Marks : 60*

Note :– Attempt all questions.

SECTION–A

1. Attempt all parts of the following : $8\times1=8$

(a) Find the order and degree of the differential equation :

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=\left(\frac{d^2y}{dx^2}\right)^3$$

(b) Find the particular integral of the differential equation :

$$(D^2+1)\,y=x^2$$

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(c) Show that $x = 0$ is not an ordinary point of the differential equation :

$$2x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + \left(x^2 - 4\right) y = 0$$

(d) Evaluate :

$$\int_{-1}^{1} x^2 P_2(x)\, dx$$

(e) If $f(x) = x$ is expanded in half range Fourier cosine series in (0, 2) then find the value of a_0.

(f) If $f(x) = x^3$ is expanded in fourier series in $(-\pi, \pi)$ then find a_0.

(g) Form the partial differential equation from $z = f(x^2 - y^2)$.

(h) Classify the partial differential equation :

$$2\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 3\frac{\partial^2 u}{\partial y^2} = 0$$

SECTION – B

2. Attempt any two parts of the following : 2×6=12

(a) Solve the following system of differential equations :

$$\frac{dx}{dt} + 4x + 3y = t$$

$$\frac{dy}{dt} + 2x + 5y = e^t$$

(b) Solve the following differential equation in series :

$$(1 - x^2) y'' - x y' + 4y = 0$$

(c) Given that $f(x) = x + x^2$ for $-\pi < x < \pi$, find the fourier expression of $f(x)$. Hence deduce :

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots\ldots\ldots$$

(d) Find the temperature in a bar of length 2 whose ends kept at zero and lateral surface insulated if the initial temperature is :

$$\sin\left(\frac{\pi x}{2}\right) + 3\sin\left(\frac{5\pi x}{2}\right)$$

SECTION – C

Note :– Attempt all questions. Attempt any two parts from each questions. 8×5=40

-(2n-1)

3. (a) Solve the following :

$$y'' - 2y' + 2y = e^x \cos x$$

(b) Solve the following :

$$\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} + 4x^2 y = x^4$$

by changing the independent variable.

(c) Use variation of parameters method to solve :

$$\frac{d^2y}{dx^2} + y = \tan x$$

4. (a) Prove that :

$$x J_n^{\prime} = n J_n - x J_{n+1}$$

(b) Prove that :

$$n P_n(x) = x P_n^{\prime}(x) - P_{n-1}^{\prime}(x)$$

(c) Prove that :

$$P_n(x) = \frac{1}{2^n \underline{|n}} \frac{d^n}{dx^n}\left(x^2 - 1\right)^n$$

5. (a) Obtain fourier series ofthe function

$$f(x) = \begin{cases} x, & -\pi < x < 0 \\ -x, & 0 < x < \pi \end{cases}$$

and hence show that :

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots\ldots = \frac{\pi^2}{8}$$

2.

(b) Express $f(x) = \frac{\pi - x}{2}$ in a fourier series in the interval $0 < x < 2\pi$. Deduce that :

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots\ldots\ldots$$

(c) Find a series of cosine of multiple of x which will represent f (x) in $(0, \pi)$ where :

$$f(x) = \begin{cases} 0, & 0 < x < \frac{\pi}{2} \\ \frac{\pi}{2}, & \frac{\pi}{2} < x < \pi \end{cases}$$

Deduce that :

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots\ldots = \frac{\pi}{4}$$

6. (a) Solve :

$$\frac{\partial^3 z}{\partial x^3} - \frac{\partial^3 z}{\partial y^3} = x^3 y^3$$

(b) Solve :

$$(D + 1)(D + D' - 1)z = \sin(x + 2y)$$

(c) Using the method of separation of variables, solve :

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$$

where $u(x, 0) = 6e^{-3x}$.

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