: 353.   BAS 3301

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Following Paper ID and Roll No. to be filled in your Answer Book.

PAPER ID : 39912   Roll No. | 2 | 0 | 4 | 3 | 2 | 2 | 4 |

B. Tech. Examination 2022-23

(Odd Semester)

COMPLEX ANALYSIS AND INTEGRAL TRANSFORMS

Time : Three Hours]   *[Maximum Marks : 60*

Note :– Attempt all questions.

SECTION–A

1. Attempt all parts of the following : $8 \times 1 = 8$

    (a) Define a harmonic function.

    (b) Write C-R equations in polar form.

    (c) Define the singularity of a function.

    (d) State the residue theorem.

    (e) Find the Laplace transform of $T^{-1/2}$.

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(f)　Find the inverse Laplace transform of $\dfrac{1}{2s-3}$.

(g)　State the fourier integral theorem.

(h)　Find the z-transform of unit impulse function.

SECTION – B

2.　Attempt any two parts of the following :　　2×6=12

(a)　Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic. Find a function v such that $f(z) = u + iv$ is analytic. Also express $f(z)$ in terms of z.

(b)　Using contour integration, evaluate the real integral :

$$\int_0^{2\pi} \frac{1}{5 - 3\cos\theta}\, d\theta.$$

(c)　Using Laplace transform method, solve

$$y'' + y = \sin 3t$$
$$y(0) = 0,\ y'(0) = 0$$

(d)　Solve the difference equation :

$$6\, y_{k+2} - y_{k+1} - y_k = 0,$$
$$y(0) = 0,\ y(1) = 1$$

by Z-transform.

SECTION – C

Note :– Attempt all questions. Attempt any two parts from each question.   5×8=40

3. (a) Show that the function $e^x (\cos y + i \sin y)$ is an analytic function. Find its derivative.

   (b) Use Cauchy's integral formula to evaluate :

   $$\int_C \frac{Z}{(z^2 - 3z + 2)} dz$$

   where C is the circle $|z - 2| = \dfrac{1}{2}$.

   (c) Evaluate :

   $$\int_0^{1+i} \left(x^2 - iy \right) dz$$

   along the path $y = x^2$.

4. (a) Find the Taylor's series expansion of the complex variable function

   $$f(z) = \frac{z+1}{(z-3)(z-4)}$$

   about $z = 2$.

   (b) Expand :

   $$f(z) = \frac{1}{(z+1)(z+3)}$$

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in laurent series valid for :

(i) $1 < |z| < 3$

(ii) $|z| > 3$

(c) Using residue theorem, evaluate :

$$\int_C \frac{4 - 3z}{z(z-1)(z-2)} \, dz$$

where C is the circle $|z| = \frac{3}{2}$.

5. (a) Find the Laplace transform of $\dfrac{e^{-t} \sin t}{t}$.

(b) Find the inverse Laplace transform of $\dfrac{s^2 + 3}{s(s^2 + 9)}$.

(c) Use laplace transform to solve :

$$\frac{dx}{dt} + y = 0 \quad \text{and} \quad \frac{dy}{dt} - x = 0$$

under the condition x (0) = 1, y (0) = 0.

6. (a) Using Fourier integral representation, show that :

$$\int_0^\infty \frac{\cos(\lambda x)}{(1 + \lambda^2)} \, d\lambda = \frac{\pi}{2} e^{-x} \quad (x > 0)$$

(b) Find the Fourier cosine transform of $f(x) = e^{-ax}$.

(c) Find the inverse Z-transform of :

$$\frac{1}{(z-2)(z-3)}, \quad |z| > 3$$

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$2 = 1$

$(2 - 1)$