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PAPER ID : 49908 Roll No. | 1 | 2 | 3 | 0 | 4 | 3 | 2 | 0 | 1 | 1 |

B. Tech. Examination, 2024-25

(Odd Semester)

COMPLEX ANALYSIS AND INTEGRAL TRANSFORMS

Time : Three Hours] *[Maximum Marks : 60*

Note :– Attempt all questions.

SECTION–A

1. Attempt all parts of the following : 8×1=8

   (a) Write C-R equations in Cartesian form.

   (b) Define harmonic function.

   (c) Write Cauchy integral formula for an analytic function.

   (d) Find the singularity for $F(z) = \frac{1}{z}$.

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(e) Find the Laplace transform of $f(t) = e^{-2t}$.

(f) Find $L^{-1}(1/s)$.

(g) Define Z-transform.

(h) Define fourier sine transform.

SECTION – B

2. Attempt any two parts of the following : 2×6=12

(a) Show that the function defined by $f(z) = \sqrt{|xy|}$ satisfy C-R equations at the origin but not analytic at that point.

(b) Evaluate :

$$\int_0^{1+i} \left(x - y + ix^2\right) dZ$$

(i) Along the straight line from z = 0 to z = 1 + i.

(ii) Along the imaginary axis from z = 0 to z = i and then along a line parallel to real axis from z = i to z = 1 + i.

(c) Find the inverse laplace transform of :

$$\log\left(1 + \frac{1}{s^2}\right)$$

(d) Find fourier transform of :

$$f(x) = \begin{cases} 1 & |x| \le a \\ 0 & |x| > a \end{cases}$$

hence evaluate :

$$\int_{-\infty}^{\infty} \frac{\sin \lambda a \cos \lambda x}{\lambda} d\lambda$$

SECTION – C

Note :– Attempt all questions. Attempt any two parts from each questions. 8×5=40

3. (a) If $u = e^x(x \cos y - y \sin y)$ is a harmonic function find an analytic function $f(z) = u + iv$ such that $f(1) = e$.

(b) Find the bilinear transformation which maps that points z = 1, i, –1 in to the points w = i, 0, –i. Hence find the image of $|z| < 1$. Where C is the circle $|z| = 3$

(c) Evaluate :

$$\oint_C \frac{e^z}{(z+1)^2} dz$$

where C is the circle $|z - 1| = 3$.

(b) Use Cauchy's Integral formula to Evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$

4. (a) Evaluate the integral using Cauchy integral formula :

$$\int_C \frac{4-3z}{z\,(z-1)\,(z-2)}\,dz$$

where C is the circle $|z| = 3/2$.

(b) Find the Taylor's and Laurent's series which represent the function :

$$\frac{z^2-1}{(z+2)\,(z+3)}$$

when :

(i) $|z| < 2$

(ii) $2 < |z| < 3$

(c) Find two bilinear transformation where fixed points are 1 and 2.

Evaluate $\int_c \frac{12z-7}{(z-1)^2(2z+3)}\,dz$ where C is the circle $|z|=2$

5. (a) Solve the simultaneous equations using Laplace transformation :

$$\frac{d^2x}{dt^2} + 5\frac{dy}{dt} - x = t,\quad 2\frac{dx}{dt} - \frac{d^2y}{dt^2} + 4y = 2$$

given that when :

$$t = 0,\ x = 0,\ y = 0,\ \frac{dx}{dt} = 0,\ \frac{dy}{dt} = 0$$

(b) Use convolution theorem to evaluate :

$$L^{-1}\left\{\frac{s}{(s^2+4)^2}\right\}$$

(c) Find $L[f(t)]$ if :

$$F(t) = \begin{cases} (t-1)^2 & t > 1 \\ 0 & 0 < t < 1 \end{cases}$$

6. (a) Solve by Z-transform the difference equation :

$$y_{k+2} + 6\,y_{k+1} + 9\,y_k = 2^k;\ (y_0 = y_1 = 0)$$

(b) Using Fourier sine integral, show that :

$$\int_0^\infty \left(\frac{1-\cos\pi\lambda}{\lambda}\right)\sin(x\lambda)\,d\lambda = \begin{cases} \pi/2 & 0 < x < \pi \\ 0 & x > \pi \end{cases}$$

(c) Solve by Z transform :

$$y_{k+1} + y_k = 1 \text{ if } y_0 = 0$$

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