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Following Paper ID and Roll No. to be filled in your Answer Book.

PAPER ID : 43204 Roll No.

B. Tech. Examination, 2024-25

(Odd Semester)

DISCRETE MATHEMATICS

Time : Three Hours] *[Maximum Marks : 60*

Note :– Attempt all questions.

SECTION – A

1. Attempt all parts of the following : $8\times1=8$

   (a) Describe whether or not following are functions defined from A to B :

   $$A=\{1,2,3,4,5\} \text{ and } B=\{a,b,c,d,e\}$$

   (i) $h=\{(5,a),(1,e),(4,b),(3,c),(2,d)\}$

   (ii) $g=\{(1,e),(5,d),(3,a),(2,b),(1,d),(4,a)\}$

   (b) Determine number of arrangements of the letters of the word "COMMITTEE".

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(c) Given $P = \{2, 3, 4, 5, 6\}$, state the truth value of the statement $(\exists\ x \in P)\ (x + 3 = 10)$.

(d) Find converse, inverse and contrapositive of the following statement. If $x + 5 = 8$ then $x = 3$.

(e) Find the power set of S where :

$$S = \{\{\phi\}, 1, \{2, 3\}\}$$

(f) If A and B are sets, then find the values of :

$$(A \cap B) \cup (A \cap \sim B)$$

using set identities.

(g) Let $S = \{(x, 3), (y, 2), (z, 3)\}$ and $T = \{(w, 2), (x, 4), (y, 3)\}$ be the multisets. Find :

(i) $S \cup T$

(ii) $S - T$

(h) Define modular lattice with an example.

SECTION – B

2. Attempt any two parts of the following : 2×6=12

(a) Let $G = \{1, -1, i, -i\}$ with the operation of ordinary multiplication on G be an algebraic structure, where $i = \sqrt{-1}$:

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(b)

(i) Determine whether G is abelian.

(ii) Determine the order of each element in G.

(iii) Determine whether G is a cyclic group, if G is a cyclic group, then determine the generator/generators of the group G.

(iv) Determine a subgroup of the group G.

(b) From the following Hasse's diagram determine the :

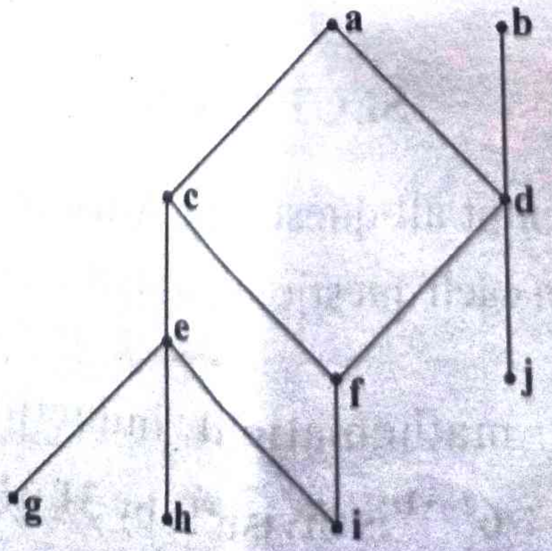
(i) Maximal elements

(ii) Minimal elements

(iii) Greatest element

(iv) $e \vee f$ (Supremum of e and f)

(v) Is this a lattice, give reason for the same?



(c) Differentiate between propositional logic and predicate logic and change the following statements in quantified expressions of predicate logic :

(i) Every student is not intelligent.

(ii) All real numberls are not rational number.

(iii) Let Q (x, y, z) : x + y = z, What as the truth values of the following statement $(\forall x)(\forall y)(\exists z)\, Q(x, y, z)$.

(d) If $f : A \rightarrow B$ and $g : B \rightarrow C$ are both bijective functions then show that :

(i) $gof : A \rightarrow C$ is also one-one.

(ii) $gof : A \rightarrow C$ is also onto.

(iii) $(gof)^{-1} : C \rightarrow A = f^{-1} o\, g^{-1} : C \rightarrow A$

SECTION – C

Note :– Attempt all questions. Attempt any two parts from each questions. $8 \times 5 = 40$

3. (a) Use mathematical induction to prove $5^{n+2} + 6^{2n+1}$ is divisible by 31 where $n \geq 1$.

(b) In a town 45% real magazine A, 55% read magazine B, 40% read magazine C, 30% read magazines A and B, 15% read magazines B and C, 25% read C and A, 10% read all the three magazines. Using set theory :

(i) What percentage reads exactly one magazine?

(ii) What percentage reads exactly two of the magazines?

(iii) What percentage reads at most two types of magazines?

(iv) What percentage does not read any magazine?

(c) Show that the relation $R = \{(a, b) : a - b = \text{even integer}, \forall\ a, b \in Z\}$ i.e. $a\,R\,b \equiv a - b =$ even integer, is an equivalence relation. Also determine whether it is a antisymmetric relation or not?

4. (a) Using composition table prove that the set $G = \{1, 2, 3, 4, 5, 6\}$ is a finite abelian group of order 6 w.r.t. multiplication modulo (X_6) operation.

(b) Define the following :

(i) Rings

(ii) Fields

(c) Draw the Hasse's diagram of D_{105} and check whether it is a complemented lattice or not, give reason for the same.

5. (a) Prove that :

$$(P \rightarrow Q) \wedge (R \rightarrow Q) \Leftrightarrow (P \vee R) \rightarrow Q$$

without using truth table (symbol $\Leftrightarrow$ means equivalent)

(b) (i) Draw the truth table of the following compound statement :

$$(\neg P \vee Q) \leftrightarrow (Q \rightarrow R).$$

(ii) Determine the converse, inverse and contrapositive of the statement :

"If today is Sunday, then I will wash the car"

(c) Using pigeon hole principle to find the minimum number of integers to be selected from the set $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ so that :

(i) The sum of two of the integers is even.

(ii) The difference of two of the n integers is 5.

6. (a) Solve the recurrence relation :

$$a_n + 5a_{n-1} + 6a_{n-2} = 42 \cdot 4^n$$

where, $a_0 = 1$ and $a_1 = -2$ by finding the homogeneous and particular solution.

(b) Using the method of generating function, solve the recurrence relation $a_n = 7a_{n-1} + 10a_{n-2}$, for $n \geq 0$ and $a_0 = 3$ and $a_1 = 3$.

(c) Prove that validity of the following argument. It is not sunny this afternoon and it is colder than yesterday. We will go swimming only if it is sunny. If we do not go swimming, we will play basketball. if we play basketball, we will go home early. Therefore "we will go home early"?

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