BAS 3101

No. of Printed Pages : 04

Following Paper ID and Roll No. to be filled in your Answer Book.

PAPER ID : 39901 **Roll No.** | | | | | | | | | | |

B. Tech. Examination - 2019

(Odd Semester)

MATRICES AND CALCULUS

Time : Three Hours] ***[Maximum Marks : 60***

Note :– Attempt all questions.

SECTION – A

1. Attempt all parts of the following : 8×1=8

(a) Show that the matrix :

$$A = \begin{bmatrix} 2 & 3-4i \\ 3+4i & 2 \end{bmatrix}$$

is Hermitian.

(b) Define elementary matrix.

(c) Find nth derivative of

$$(e^{3x} + e^{-3x})$$

$3^n e^{3x} + (-3)^n e^{-3x}$

(d) If

$$u = x\,\phi\left(\frac{y}{x}\right)$$

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u$

then find the value of

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$$

/ P. T. O.

(e) Find grad ϕ when $\phi = 3x^2 y - y^3 z^2$ at the point $(1, -2, -1)$.

$-12i - 9j - 16k$

(f) State Gauss divergence theorem.

(g) Write the formula of area of the region bounded by the curves $y = f_1(x)$, $y = f_2(x)$ and lines $x = a$, $x = b$ by double integration.

(h) Find the value of $\Gamma\left(\frac{1}{2}\right)$ $\quad -2\sqrt{\pi}$

SECTION – B

2. Attempt any two parts of the following : 2×6=12

(a) Find the eigen values and eigen vectors of the matrix :

$0, 3, 15$

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \qquad \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$$

(b) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{m}\right)^m$ prove that

$+xy, -m^2y = 0$

$$x^2 y_{n+2} + (2n+1) \times y_{n+1} + (n^2 + m^2) y_n = 0$$

(c) Prove that : $B(\ell, m) = \frac{\Gamma e \, \Gamma m}{\Gamma e + m}$

(d) Evaluate $\oint_C \vec{F} \cdot d\vec{r}$ by Stoke's theorem where $\vec{F} = y^2 i + x^2 j - (x+z) k$ and C is the boundary of the triangle with vertices at $(0, 0, 0)$, $(1, 0, 0)$ and $(1, 1, 0)$.

SECTION – C

Note :– Attempt all questions. Attempt any two parts from each questions. 5×8=40

3. (a) Find the rank of matrix :

$$A = \begin{bmatrix} 2 & 3 & 4 & -1 \\ 5 & 2 & 0 & -1 \\ -4 & 5 & 12 & -1 \end{bmatrix}$$

$\rho(A) = 2$

(b) Apply the matrix method to solve the system of equations : $x + 2y - z = 3$, $3x - y + 2z = 1$, $2x - 2y + 3z = 2$ and $x - y + z = -1$.

$x = -1$
$y = 4$
$z = 4$

(c) Verify Cayley-Hamilton theorem for the matrix :

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$ Hence find A^{-1}.

$\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$

$A^{-1} = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} & \frac{3}{4} \end{bmatrix}$

4. (a) Prove that the functions $u = x + y - z$, $v = x - y + z$, $w = x^2 + y^2 + z^2 - 2yz$ are not independent (functionally dependent) and then find relation between them.

$u^2 + v^2 = 2w$

(b) Expand $e^x \sin y$ in powers of x and y as far as terms of the third degree.

$y + xy + \frac{x^2 y}{2} - \frac{y^3}{6}$

(c) State Euler's theorem and hence prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 2\tan u$ where

$$u = \sin^{-1}\left(\frac{x^3 + y^3 + z^3}{ax + by + cz}\right)$$

5. (a) Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point P (1, 2, 3) in the direction of the line P θ where θ (5, 0, 4).

(Ch-2, 1)

(b) Prove that : $\vec{F} = (y^2 - z^2 + 3yz - 2x)\,i + (3xz + 2xy)\,j + (3xy - 2xz + 2z)\,k$ is both solenoidal and irrotational.

(c) If $\bar{F} = 4xz\,i - y^2\,j + yz\,k$, then evaluate :

$$\iint_S \bar{F} \cdot \hat{n}\, ds$$

by using Gauss-divergence theorem. Where S is surface taken over the cube bounded by planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

$\frac{3}{2}$

6. (a) Change the order of the integration in :

$\pi \frac{9}{4}$

$$\int_0^a \int_y^a \frac{x\,dx\,dy}{(x^2 + y^2)}$$ and hence evaluate :

(b) Prove that :

$$\frac{B(m, n+1)}{n} = \frac{B(m+1, n)}{m} = \frac{B(m, n)}{m+n}$$

(c) Evaluate $\iiint_R (x + y + z)\,dx\,dy\,dz$ where $R : 0 \le x \le 1, 0 \le y \le 2$ and $2 \le z \le 3$.