BAS 3101

S.No. : 839

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Following Paper ID and Roll No. to be filled in your Answer Book.											
PAPER ID : 39901	Roll No.	1	2	2	0	4	3	2	6	2	5

B. Tech. Examination 2022-23

(Odd Semester)

MATRICES AND CALCULUS

Time : Three Hours] *[Maximum Marks : 60*

Note :– Attempt all questions.

SECTION – A

1. Attempt all parts of the following : $8 \times 1 = 8$

(a) Define unitary matrix.

(b) For which value of 'K' the rank of the matrix :

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 3 \\ 4 & 3 & 6 \end{bmatrix} \text{ is } 2$$

(c) Find the nth derivative of $\log(3x^2 + x^3)$.

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(d) If $u = \dfrac{x+y}{\sqrt{x}+\sqrt{y}}$:

find :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$$

(e) Evaluate :

$$\int_0^1 \int_0^x e^{y/x}\, dx\, dy$$

(f) Evaluate :

$$\Gamma\left(-\frac{3}{2}\right)$$

(g) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then find $\vec{\nabla}\cdot\vec{r}$.

(h) State Stokes theorem.

SECTION – B

2. Attempt any two parts of the following : 2×6=12

(a) Find the eigen value and eigen vectors of the matrix :

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

(b) If $y=\left(x+\sqrt{1+x^2}\right)^m$, prove that :

$$(1+x^2)\,y_{n+2}+(2n+1)\,x\,y_{n+1}+(n^2-m^2)\,y_n=0$$

and hence find $y_n(0)$.

(c) Find the mass of an octant of the ellipsoid :

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

the density at any point being $\rho=kxyz$.

(d) Verify the Gauss divergence theorem for :

$$\bar{F}=\left(x^2-yz\right)\hat{i}+\left(y^2-zx\right)\hat{j}+\left(z^2-xy\right)\hat{k}$$

taken over the rectangular parallelopiped $0\le x\le a,\ 0\le y\le b,\ 0\le z\le c$.

SECTION – C

Note :– Attempt all questions. Attempt any two parts from each question. 5×8=40

3. (a) Find the rank of the matrix :

$$A=\begin{bmatrix}1 & 3 & 4 & 2\\ 2 & -1 & 3 & 2\\ 3 & -5 & 2 & 2\\ 6 & -3 & 8 & 6\end{bmatrix}$$

by reducing it to normal form.

(b) Investigate the values of λ and μ so that the equations :

$$2x + 3y + 5z = 9$$
$$7x + 3y - 2z = 8$$
$$2x + 3y + \lambda z = \mu$$

have :

(i) No solution

(ii) A unique solution

(iii) An infinite number of solution

(c) Verify Cayley-Hamilton theorem for the matrix :

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

and hence find A^{-1}.

4. (a) If $u = xy + yz + zx$, $V = x^2 + y^2 + z^2$ and $w = x + y + z$, determine whether there is a functional relationship between u, v, w and if so, find it.

(b) Examine $f(x, y) = x^3 + y^3 - 3axy$ for maximum and minimum values.

(c) If $x^x y^y z^z = c$, show that at :

$$x = y = z, \frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$$

5. (a) Change the order of integration in :

$$\int_0^2 \int_{\sqrt{4-x^2}}^{4-x} f(x, y)\, dy\, dx$$

(b) Prove that :

$$\beta(m, n) = \frac{\Gamma m \, \Gamma n}{\Gamma(m+n)}$$

(c) Evaluate :

$$\int\int\int_R (x - 2y + z)\, dx\, dy\, dz$$

where :

$$R : 0 \le x \le 1, 0 \le y \le x^2, 0 \le z \le x + y$$

6. (a) Use Green's theorem, to evaluate :

$$\int_C \left(x^2 + xy\right) dx + \left(x^2 + y^2\right) dy$$

where C is the square formed by the lines $y = \pm 1, x = \pm 1$.

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(b) If $\vec{A} = (x - y)\,\hat{i} + (x + y)\,\hat{j}$, evaluate :

$$\oint_C \vec{A} \cdot d\vec{r}$$

around the curve C consisting of $y = x^2$ and $y^2 = x$.

(c) Determine the constant a and b such that the curl of vector :

$$\vec{A} = (2xy + 3yz)\hat{i} + (x^2 + axz - 4z^2)\,\hat{j} - (3xy + byz)\,\hat{k} \text{ is zero}$$

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