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Following Paper ID and Roll.No. to be filled in your Answer Book.

Paper ID : 49901	Roll No.										

B.Tech. Examination -2023-24

(Odd Semester)

MATRICES AND CALCULUS

Time : Three Hours] **[Maximum Marks : 60**

Note :– Attemtp all questions.

SECTION-A

1. Attempt each part in this section. Each part carry equal marks. 8 x 1=8

(a) Define orthogonal matrix.

(b) Define rank of a matrix.

(c) Find the n^{th} derivative of $\log x^2$.

(d) State Euler's theorem on homogeneous function.

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(e) Evaluate—

$$\int_0^a \int_0^x xy \, dy \, dx$$

(f) Evaluate—

$$\int_0^\infty \sqrt{x} \, e^{-x} \, dx$$

(g) Find the normal vector to the surface $z=2xy$ at (2, 1,4).

(h) What is surface integral of a vector function f over the surface S.

SECTION-B

2. Attempt any two parts in this section. Each part carry equal marks. 2 x 6 = 12

(a) Verify Cayley Hamilton theorem for the matrix $\begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$. Hence find A^{-1}.

(b) Find the value of the nth derivative of $y=e^{m\sin^{-1}x}$ for $x=0$.

(c) Find the shortest distance from the point (1,2,–1) to the sphere $x^2+y^2+z^2=24$.

(d) Verify Stoke's theorem for the function $\vec{F}=x^2\hat{i}-xy\hat{j}$ integrated round the square in the plane $z=0$ and bounded by the lines $x=0$, $y=0$, $x=a$, $y=a$.

SECTION-C

3. Attempt any two parts from each questions. Each part carry equal marks. 5 x 8 = 40

(a) Find the rank of the matrix—

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$

by reducing it to normal form.

(b) For what value of λ the equations

$$x+y+z=1$$
$$x+2y+4z=\lambda$$
$$x+4y+10z=\lambda^2$$

has a solution andsolve them compeletly in each case.

(c) Find the eigen values and corresponding eigen vectors of the matrix—

$$A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$$

4. (a) Expand $e^x \sin y$ in powers of x and y upto terms of third degree.

(b) If $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_3 x_1}{x_2}$, $y_3 = \frac{x_1 x_2}{x_3}$ then show that the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 is 4.

(c) If $V = \left(x^2 + y^2 + z^2\right)^{m/2}$ then find the value of $m (m \neq 0)$ which will make $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$

5. (a) Evaluate $\int_0^2 \int_1^{e^x} dx\, dy$ by changing the order of integration.

(b) Evaluate—

$$\int_0^{\log 2} \int_0^{x} \int_0^{x+y} e^{x+y+z}\, dx\, dy\, dz$$

(c) Prove that—

$$\Gamma n \, \Gamma\left(n+\frac{1}{2}\right) 2^{2n-1} = \sqrt{\pi}\, \Gamma 2n$$

where n is posive.

6. (a) Find the directional derivative of $\phi(x, y, z) = x^2 yz + 4xz^2$ at (1, –2, 1) in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$.

(b) If $\vec{A} = (3x^2\ 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$ evaluate the line integral $\oint \vec{A}.d\vec{r}$ from (0,0,0) to (1,1,1) along the curve $c: x=t,\ y=t^2,\ z=t^3$

(c) Using Green's theorem evalute $\int_c \left(x^2 y\, dx + x^2\, dy\right)$, where c is the boundary described counter clockwise of the triangle with vertices (0,0), (1,0), (1,1)